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INELASTIC BUCKLING OF RECTANGULAR SANDWICH PLATES

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Abstract—An analytical study of buckling of rectangular sandwich plates, stressed uniaxially by uniform shortening beyond the elastic limits of component materials, is presented. The analysis is based on the inelastic behavior according to both the J_2-incremental and J_2-deformation theories of plasticity. Taking the loaded edges as simply supported, the governing equations are solved for (a) plates simply supported on all four sides, and (b) plates simply supported on three sides and free on the fourth side. The theory gives rise to two sets of boundary conditions, each representing "simple support". It is shown that these alternative choices lead to significantly different predictions of the buckling load in the case of sandwich plates simply supported on three sides and free on the fourth, unloaded side. The presented analysis can be specialized to elastic buckling of sandwich plates, and also to elastic or plastic buckling of homogeneous (one-material) plates with or without the transverse shear effects.

NOTATION

B', C', D', F'	moduli used in the stress–strain relations. Subscripts f and c refer, respectively, to the moduli of the facing and core materials
a, b	length and width of a rectangular sandwich plate
B, C, D, F	composite moduli for a sandwich plate
E	Young's modulus of elasticity
E_s, E_t	secant, tangent moduli
$e = E/E_s - 1$	quantity used in deformation theory
G	$= k\,F_c h^3$
h	core thickness
J_2	second invariant of the deviatoric stress tensor ($= \frac{1}{2} s_{ij} s_{ij}$)
k	shear correction factor
M_{xx}, M_{yy}, M_{xy}	moment stress resultants/unit length of the plate due to buckling
m, n	number of half-waves in a buckled plate in longitudinal and transverse directions, respectively
P_{cr}	buckling load/unit edge length
Q_x, Q_y	transverse shear stress resultants/unit length of the plate due to buckling
$\bar{u}, \bar{v}, \bar{w}$	x, y, z displacement components
α	$= m\pi/a$
$d\varepsilon_{ij}$	components of the infinitesimal strain tensor due to buckling
ε	$= F/G$, a parameter related to the ratio of transverse shear moduli of the core and facings
κ	$= P_{cr}h^2/F$, buckling parameter
λ	$= E/E_t$, ratio of Young's modulus to tangent modulus in uniaxial compression test
ν	Poisson's ratio
σ_{cr}	$= P_{cr}/2t =$ nominal buckling stress
$d\sigma_{ij}$	increment of stress components due to buckling
σ_1, ε_1	stress–strain in uniaxial compression
$\phi(x, y)$	component of rotation of the normal to middle plane about y-axis
$\psi(x, y)$	component of rotation of the normal to middle plane about x-axis
$\beta(x, y), \theta(x, y)$	variables, alternative to ϕ and ψ, respectively
η	$= \alpha y$, non-dimensional variable in place of y

1. INTRODUCTION

It is well recognized that the analysis and design of sandwich structural components are more complex and varied than those of homogeneous ones. A typical example illustrating this difficulty lies in the analysis of sandwich plates, consisting of metal facings and a core of a different material. Since usually the core is of a material with a shear modulus much lower than that of the facing plates, consideration of the effect of transverse shear deformations is of fundamental importance in structural analysis of sandwich plates. This is in contrast to the theory of homogeneous plates, where such deformations are of secondary importance and are usually neglected. Moreover, since facings are relatively thin, stability considerations also become important in determining the behavior of the sandwich plates. Thus, the analysis of sandwich plates calls for a higher-order theory than that normally used for homogeneous plates.

Reissner (1945) was the first to devise an engineering theory which took into account the effect of transverse shear deformations on the elastic bending behavior of plates in a simple manner. The fundamental variables in his theory are the two transverse shears (V_x and V_y) and the transverse deflection w, leading to three simultaneous partial differential equations in these variables. With some additional assumptions, Reissner extended his theory to "finite" elastic deflections of sandwich plates, and also to their elastic buckling (Reissner, 1948).

The work of Libove and Batdorf (1947) is concerned especially with the formulation of a theory applicable to the elastic bending and buckling analysis of sandwich plates with orthotropic or isotropic cores. They used basically the same variables as Reissner (1945), but their approach is based on kinematic rather than on stress assumptions. This approach enables them to present an alternative derivation of the governing equations and boundary conditions, using the principle of stationary potential energy. Hoff (1950) presented a somewhat simpler theory of sandwich plates by using the in-plane facing displacement components, u, v, and the out-of-plane deflection, w, as his basic variables, and obtained governing equations using the energy approach.

The theory of Libove and Batdorf (1947) was applied to the buckling of sandwich plates, both in the elastic and inelastic ranges, by Seide and Stowell (1948). They obtained the buckling stresses for flat, rectangular, simply supported sandwich plates loaded in uniaxial compression, and compared their results with available experimental results. The stress–strain relations used for the inelastic range were according to the J_2-deformation theory of plasticity.

Since the governing equations for the buckling analysis are, in general, a set of simultaneous partial differential equations, rigorous solutions can be obtained only for cases with simple boundary conditions, for example, simply supported conditions. This has led to the situation that, whereas theories abound (Chang *et al.*, 1967; Plantema, 1966; Howard, 1969), solutions of practically important problems (other than the buckling of rectangular sandwich plates simply supported on all sides) are lacking. A notable exception is the work of Bijlaard (1951a,b), in which the author gives results for several cases of boundary conditions of rectangular sandwich plates, and also deals with the inelastic instability of these plates on the basis of the J_2-deformation theory of plasticity. However, again, these results are rigorous only for the simply supported plates.

One of the objectives of the present investigation is to present a theory of buckling of sandwich plates which is simpler than the aforementioned theories in the choice of basic variables and in the resulting system of differential equations. Instead of the shears (V_x, V_y), the rotations (θ,ϕ) are taken as the basic dependent variables. The general theory is that reported in Shrivastava (1979), and is applied in this work to the inelastic bifurcation buckling of sandwich plates, using both the incremental and deformation theories of plasticity. The inelastic constitutive relations for both theories are based on the Mises yield condition ($J_2 = c$) and on Shanley's concept of bifurcation buckling under continued loading ($dJ_2 > 0$).

Although the incremental theory is considered as the correct phenomenological theory of plasticity, it is well known that, when applied to bifurcation buckling of flat homogeneous plates, it predicts results which are unrealistically higher than the experimental values. On the other hand, the deformation theory of plasticity gives results which are in reasonable (and safe-side) agreement with experiments. Hence, despite its ostensibly "weak" theoretical basis, investigators have continued to use the deformation theory in analyzing plastic buckling of plate and shell structures.

In this paper, results are derived according to both plasticity theories, so that a comparison can be made to see to what extent the results diverge in the case of sandwich plates. Rigorous analysis is carried out for buckling of flat rectangular sandwich plates loaded by uniform shortening in the longitudinal direction. The loaded (x = constant) edges are considered simply supported, while the unloaded (y = constant) edges are allowed to be subjected to various boundary conditions. Although the core material may be considered to be in the inelastic range, together with the facings, the presented analysis is restricted to isotropic elastic behavior of the core. Two important cases of buckling of long sandwich plates (with longitudinal edges unloaded) are considered: (1) both longitudinal edges are simply supported; and (2) one longitudinal edge is simply supported and the other is free. Moreover, separate analyses are made for the two choices of simple-support conditions on the longitudinal edges, corresponding to whether the twisting angle, ϕ, or the twisting moment, M_{xy}, is taken to be zero. It is shown that the differences between the buckling stresses so obtained become very significant in the case of plates supported on three edges and free on the fourth edge; such differences do not arise in the case of homogeneous plates.

Although the analysis is exact, closed-form solution is obtainable only in the simplest case. Thus, the use of a computer becomes essential in solving the characteristic equation for obtaining numerical values of the buckling loads. These are given for sandwich plates consisting of 24S-T3 aluminum alloy facings and balsa wood core. The validity of the analytical procedure and the verification of the computer program were carried out by considering the degenerated case of the buckling of homogeneous plates. The results thus obtained agree with those of Shrivastava (1979). It may be noted that the present analysis assumes the core to be sufficiently strong in the thickness, z, direction, that wrinkling instability of facings does not occur. This type of instability in the elastic range has been considered by several investigators, e.g. Yusuff (1955).

2. CONSTITUTIVE RELATIONS

Figure 1 shows the plate and the coordinate system used for the buckling analysis. The plate is taken to be a perfect plane, with facings of uniform and equal thickness and located symmetrically with respect to the middle plane $z = 0$. The core thickness is denoted by h, and that of facings by t.

The common assumptions of the two plasticity theories are taken to be: (1) that the strains are small, with the plastic part being volume-preserving, and (2) that the material is loaded from a virgin state, remains isotropic, and obeys the Mises yield condition, namely $J_2 = \sigma^2/3$, where σ is the (true) stress recorded in the uniaxial compression test of the virgin material in the plastic range.

The prebuckling state of stress is assumed to be uniaxially compressive ($\sigma^0_{ij} = 0$ except $\sigma^0_{xx} = -\sigma$), corresponding to a uniform shortening of the plate in the x (longitudinal) direction (thus ignoring other presumably small prebuckling stresses arising out of displacement compatibility at perfectly bonded sandwich interfaces). When the plate buckles, the state of stress is changed from σ^0_{ij} to $\sigma^0_{ij}+\mathrm{d}\sigma_{ij}$, $\mathrm{d}\sigma_{ij}$ being the increment in stress components due to buckling. Invoking the usual engineering approximation that $\mathrm{d}\sigma_{zz} = 0$, the relationship between $\mathrm{d}\sigma_{ij}$ and the increments of strain $\mathrm{d}\varepsilon_{ij}$, for the case of loading ($dJ_2 > 0$), are taken from Shrivastava (1979) as

$$\mathrm{d}\sigma_{11} = B'\,\mathrm{d}\varepsilon_{11}+C'\,\mathrm{d}\varepsilon_{22}, \quad \mathrm{d}\sigma_{22} = C'\,\mathrm{d}\varepsilon_{11}+D'\,\mathrm{d}\varepsilon_{22}, \quad \mathrm{d}\sigma_{ij} = 2F'\,\mathrm{d}\varepsilon_{ij}\,(i \neq j), \tag{1}$$

where

$$\begin{aligned} B' &= E(\lambda+3+3e)/[\lambda(5-4\nu+3e)-(1-2\nu)^2], \\ C' &= 2E(\lambda-1+2\nu)/[\lambda(5-4\nu+3e)-(1-2\nu)^2], \\ D' &= 4E\lambda/[\lambda(5-4\nu+3e)-(1-2\nu)^2], \\ F' &= E/(2+2\nu+3e). \end{aligned} \tag{2}$$

The parameters $\lambda = E/E_t$ and $e = (E/E_s)-1$ are considered obtainable from a uniaxial compression test. E_t and E_s denote, respectively, the tangent and secant moduli at the stress level σ.

The relations (2), as they are written, are derived according to the J_2-deformation theory of plasticity. However, they include relations according to the J_2-incremental theory of plasticity, and also those for the linear isotropic elastic case. The former are obtained by substituting $e = 0$ in eqns (2), and the latter by putting $e = 0$ and $\lambda = 1$. Thus, the analysis based on the deformation theory of plasticity can be immediately specialized to that for the incremental plasticity theory and elastic theory. In the analysis that follows, subscripts or superscripts f and c will be used to distinguish facings and core properties, respectively.

3. GOVERNING EQUATIONS AND THEIR SOLUTION

In order to account for the effect of transverse shear deformations, the kinematic hypothesis of Kirchhoff in conventional plate theory is modified by assuming that a normal to the undeflected middle plane remains straight, but not necessarily perpendicular to the deflected middle surface. Then, denoting by ϕ and ψ the components of rotation of the normal in the x–z and y–z planes, respectively, the displacements of the points of the plate, arising due to buckling, are taken as

$$\bar{u} = u(x,y) - z\phi(x,y),$$
$$\bar{v} = v(x,y) - z\psi(x,y),$$
$$\bar{w} = w(x,y), \tag{3}$$

where u, v, w are the displacement components of points at the middle plane $z = 0$. As is evident, the last of eqns (3) implies $d\varepsilon_{zz} = 0$, in conflict with the previous assumption of $d\sigma_{zz} = 0$. However, as in the case of homogeneous plates, this inconsistency is considered unimportant for plates which are only moderately thick, and in which the core material is sufficiently stiff in the z-direction.

The equilibrium equations and the boundary conditions, appropriate to the above kinematic assumptions, are obtained by using the principle of virtual work. The equilibrium equations pertaining to the out-of-plane buckling ($u = v = 0$, $w \neq 0$) of a plate loaded by uniform compressive load in the x-direction, are

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - P_{cr} w_{xx} = 0, \tag{4}$$

with the work-conjugate pairs of boundary conditions

$$M_{xx} = 0 \text{ or } \delta\phi = 0, \quad M_{xy} = 0 \text{ or } \delta\psi = 0, \quad Q_x = P_{cr} w_x \text{ or } \delta w = 0 \tag{5a}$$

at the edges x = constant, and

$$M_{yy} = 0 \text{ or } \delta\psi = 0, \quad M_{xy} = 0 \text{ or } \delta\phi = 0, \quad Q_y = 0 \text{ or } \delta w = 0 \tag{5b}$$

at the edges y = constant. P_{cr} is the uniform compressive load per unit width of the plate at which it begins to buckle, and is given by

$$P_{cr} = (h\sigma_c + 2t\sigma_f), \tag{6}$$

where σ_f is the stress in the facings, and σ_c that in the core at the instant of buckling.

Now, in accordance with Shanley's concept in the plastic buckling of columns, it is postulated that the plastic buckling occurs under increasing load, so that there is no unloading from plasticity when the plane plate bifurcates into a buckled shape. The relations

(2), therefore, apply throughout the thickness of the plate, and the buckling stress resultants are expressible as

$$M_{xx} = -[B\phi_x + C\psi_y],\quad M_{yy} = -[C\phi_x + D\psi_y],\quad M_{xy} = -F[\phi_y + \psi_x],$$
$$Q_x = G[-\phi + w_x]/h^2,\quad Q_y = G[-\psi + w_y]/h^2, \tag{7}$$

where

$$B = \frac{B_c' h^3}{12}\left[1 + \frac{B_f'}{B_c'} f(t)\right],\quad C = \frac{C_c' h^3}{12}\left[1 + \frac{C_f'}{C_c'} f(t)\right],$$
$$D = \frac{D_c' h^3}{12}\left[1 + \frac{D_f'}{D_c'} f(t)\right],\quad F = \frac{F_c' h^3}{12}\left[1 + \frac{F_f'}{F_c'} f(t)\right], \tag{8a}$$
$$G = kF_c' h^3,\quad \text{and}\quad f(t) = 6(t/h) + 12(t/h)^2 + 8(t/h)^3. \tag{8b}$$

The coefficients B_f', C_f', D_f' and F_f' for the facing material, and B_c', C_c', D_c' and F_c' for the core, are defined by using appropriate material constants in eqns (2), where it should be remembered that the axial compressive stress in the core is different from that in the facings. The symbol k stands for a correction factor which takes into account (contrary to the strain assumption) the non-uniform distribution of transverse shear stresses through the thickness of the plate. It can be shown that $k \approx 1$ for sandwich plates of moderate facing/core thickness ratio ($t/h \leqslant 0.25$), whereas it is well known that $k = 5/6$ for homogeneous plates. Here, we take $k = 1$.

Substitution of relations (6) into the equilibrium equations (4) leads to the following set of governing equations:

$$\left[B\frac{\partial^2\phi}{\partial x^2} + F\frac{\partial^2\phi}{\partial y^2} - \frac{G}{h^2}\phi\right] + (C+F)\frac{\partial^2\psi}{\partial x\partial y} + \frac{G}{h^2}\frac{\partial w}{\partial x} = 0,$$
$$(C+F)\frac{\partial^2\phi}{\partial y\partial x} + \left[D\frac{\partial^2\psi}{\partial y^2} + F\frac{\partial^2\psi}{\partial x^2} - \frac{G}{h^2}\psi\right] + \frac{G}{h^2}\frac{\partial w}{\partial y} = 0,$$
$$\frac{G}{h^2}\frac{\partial\phi}{\partial x} + \frac{G}{h^2}\frac{\partial\psi}{\partial y} - \left[\frac{G}{h^2}\frac{\partial^2 w}{\partial x^2} + \frac{G}{h^2}\frac{\partial^2 w}{\partial y^2} - P_{cr}\frac{\partial^2 w}{\partial x^2}\right] = 0. \tag{9}$$

The boundary conditions on stress resultants may also be expressed in terms of ϕ, ψ and w by means of eqns (5). The above partial differential equations, together with the homogeneous boundary conditions, constitute a linear symmetric eigenvalue problem for determining the eigenvalue parameter P_{cr}. We also note that eqns (9) cease to represent the eigenvalue problem when $P_{cr} = G/h^2 \approx F_c' h$. This value of P_{cr} is associated with the pure shear (crimping) instability of the core. Consequently, we restrict our attention to such sandwich plates for which $P_{cr} < G/h^2$.

It seems that in numerical solutions of these equations (not attempted here), they may prove difficult to solve accurately, on account of the differences in approximately equal quantities (e.g. between ϕ and w_x). To remedy this likelihood in future applications, we introduce the following change of variables:

$$\frac{G}{F}\left(-\phi + \frac{\partial w}{\partial x}\right) = \beta,\quad \text{or}\quad \phi = \frac{\partial w}{\partial x} - \varepsilon\beta,\quad \text{and}$$
$$\frac{G}{F}\left(-\psi + \frac{\partial w}{\partial y}\right) = \theta,\quad \text{or}\quad \psi = \frac{\partial w}{\partial y} - \varepsilon\theta, \tag{10}$$

where

$$\varepsilon = \frac{F}{G} = \left[1 + \frac{F_f'}{F_c'} f(t)\right] / 12k.$$

The governing equations (9) are then transformed into

$$\left[-\frac{\varepsilon B}{F}\frac{\partial^2 \beta}{\partial x^2} - \varepsilon\frac{\partial^2 \beta}{\partial y^2} + \frac{\beta}{h^2}\right] - \varepsilon\left[\frac{C+F}{F}\right]\frac{\partial^2 \theta}{\partial x \partial y} + \left[\frac{B}{F}\frac{\partial^3 w}{\partial x^3} + \left(2 + \frac{C}{F}\right)\frac{\partial^3 w}{\partial x \partial y^2}\right] = 0,$$

$$-\varepsilon\left[\frac{C+F}{F}\right]\frac{\partial^2 \beta}{\partial x \partial y} + \left[-\frac{\varepsilon D}{F}\frac{\partial^2 \theta}{\partial y^2} - \varepsilon\frac{\partial^2 \theta}{\partial x^2} + \frac{\theta}{h^2}\right] + \left[\frac{D}{F}\frac{\partial^3 w}{\partial y^3} + \left(2 + \frac{C}{F}\right)\frac{\partial^3 w}{\partial y \partial x^2}\right] = 0,$$

$$-\frac{\partial \beta}{\partial x} - \frac{\partial \theta}{\partial y} + \frac{P_{cr} h^2}{F}\frac{\partial^2 w}{\partial x^2} = 0. \quad (11)$$

We note that the above introduction of variables β and θ amounts to introducing the shears V_x and V_y of Reissner's (1945) theory. For a homogeneous plate one has $t = 0$ and $k = 5/6$, and ε takes on its minimum value, 0.1. The neglect of transverse shear deformations (Kirchhoff theory) entails $\varepsilon = 0$.

Separation of variables

For an analytical approach, the treatment is restricted to the boundary conditions which allow separation of variables. The conditions at the loaded edges allowing this separation are

$$M_{xx} = -\left[B\frac{\partial \phi}{\partial x} + C\frac{\partial \psi}{\partial y}\right] = 0, \quad w = 0, \quad \psi = 0 \text{ at } x = 0, a. \quad (12)$$

These boundary conditions correspond to simple support of the type in which rotation perpendicular to the edge is unrestricted ($M_{xx} = 0$) but rotation parallel to the edge is restricted (i.e. $\psi = 0$, and hence $M_{xy} \neq 0$). In practice, this type of edge constraint may be realized, for example, by welding a stiff channel to the facings.

A possible solution to eqns (9), which satisfies the above boundary conditions, is

$$\phi(x, \eta) = \Phi(\eta)\cos(\alpha x), \quad \psi(x, \eta) = \Psi(\eta)\sin(\alpha x), \quad w(x, \eta) = \frac{\bar{W}(\eta)}{\alpha}\sin(\alpha x), \quad (13)$$

where $\alpha = m\pi/a$, m is the number of half-waves in which the plate buckles, a is the length of the plate, and η is a non-dimensional coordinate defined by

$$\eta = \alpha y = \frac{m\pi y}{a}. \quad (14)$$

In terms of the transformed variables the above relations are equivalent to:

$$\beta(x, \eta) = \frac{1}{\varepsilon}[-\Phi(\eta) + \bar{W}(\eta)] \cdot \cos(\alpha x) = \bar{\beta}(\eta)\cos(\alpha x),$$

$$\theta(x, \eta) = \frac{1}{\varepsilon}\left[-\Psi(\eta) + \frac{d\bar{W}(\eta)}{d\eta}\right] \cdot \sin(\alpha x) = \bar{\theta}(\eta)\sin(\alpha x), \quad (15)$$

where

$$\bar{\beta} \equiv \bar{\beta}(\eta) = \frac{1}{\varepsilon}[-\Phi(\eta) + \bar{W}(\eta)] \text{ and } \bar{\theta} \equiv \bar{\theta}(\eta) = \frac{1}{\varepsilon}\left[-\Psi(\eta) + \frac{\mathrm{d}\bar{W}(\eta)}{\mathrm{d}\eta}\right]. \tag{16}$$

Substituting eqns (15) into eqns (11), the following ordinary differential equations with non-dimensional coefficients are obtained:

$$\left[\left(\varepsilon\frac{B}{F} + \frac{1}{h^2\alpha^2}\right)\bar{\beta} - \varepsilon\frac{\mathrm{d}^2\bar{\beta}}{\mathrm{d}\eta^2}\right] - \varepsilon\left[\frac{C+F}{F}\right]\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\eta} + \left[-\frac{B}{F}\bar{W} + \left(2 + \frac{C}{F}\right)\frac{\mathrm{d}^2\bar{W}}{\mathrm{d}\eta^2}\right] = 0,$$

$$\varepsilon\left[\frac{C+F}{F}\right]\frac{\mathrm{d}\bar{\beta}}{\mathrm{d}\eta} + \left[-\frac{\varepsilon D}{F}\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}\eta^2} + \left(\varepsilon + \frac{1}{h^2\alpha^2}\right)\bar{\theta}\right] + \left[\frac{D}{F}\frac{\mathrm{d}^3\bar{W}}{\mathrm{d}\eta^3} - \left(2 + \frac{C}{F}\right)\frac{\mathrm{d}\bar{W}}{\mathrm{d}\eta}\right] = 0,$$

$$\bar{\beta} - \frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\eta} - \frac{P_{\mathrm{cr}}h^2}{F}\bar{W} = 0. \tag{17}$$

Following the usual procedure, let the solution of eqns (17) be expressed as

$$\bar{W} = A_1 \exp(s\eta), \quad \bar{\beta} = B_1 \exp(s\eta), \quad \bar{\theta} = C_1 \exp(s\eta). \tag{18}$$

Then it follows from eqns (17) that $\gamma = s^2$ is a root of

$$\varepsilon\frac{D}{F}\gamma^3 - K_1\gamma^2 + K_2\gamma + K_3 = 0, \tag{19}$$

where

$$K_1 = \frac{\varepsilon D}{F}(1-R) + \frac{D}{Fh^2\alpha^2} - \varepsilon^2\kappa\frac{D}{F},$$

$$K_2 = \frac{D}{Fh^2\alpha^2}\left(\frac{4F}{D} + \frac{2C}{D}\right) - \varepsilon\left(\frac{RD}{F} - \frac{B}{F}\right) - \kappa\left\{\varepsilon\left(\frac{1}{h^2\alpha^2} + \frac{D}{Fh^2\alpha^2}\right) - \varepsilon^2\frac{RD}{F}\right\},$$

$$K_3 = -\frac{\varepsilon B}{F} - \frac{B}{Fh^2\alpha^2} + \kappa\left(\frac{\varepsilon^2 B}{F} + \frac{\varepsilon}{h^2\alpha^2} + \frac{\varepsilon B}{Fh^2\alpha^2} + \frac{1}{h^4\alpha^4}\right) \tag{20}$$

and

$$\kappa = \frac{P_{\mathrm{cr}}h^2}{F},$$

$$R = -\frac{B}{F} + \frac{C^2}{DF} + \frac{2C}{D}. \tag{21}$$

Solution

Let the roots of the cubic equation, eqn (19), be written as

$$\gamma_1 = -q^2, \quad \gamma_2 = (a+ib), \quad \gamma_3 = (a-ib), \tag{22}$$

where a and q^2 are real numbers. However, b can be either purely real (say positive), purely complex, or zero, depending on whether the two latter roots are complex conjugate, real and distinct, or real and equal, respectively. Then, in view of $\gamma = s^2$, we have

$$s_1 = iq, \quad s_2 = -iq,$$

$$s_3 = (c+id), \quad s_4 = -(c+id),$$

$$s_5 = (c-id), \quad s_6 = -(c-id), \tag{23}$$

where

$$c = \sqrt{\{a+(a^2+b^2)^{1/2}\}/2} \quad \text{and} \quad d = \sqrt{\{-a+(a^2+b^2)^{1/2}\}/2}. \tag{24}$$

By introducing the notation

$$\begin{aligned}
g_1 &= \frac{\sin(q\eta)}{q}, \quad g_2 = \cos(q\eta), \\
g_3 &= \sinh(c\eta)\frac{\sinh(d\eta)}{d}, \quad g_4 = \cosh(c\eta)\cosh(d\eta), \\
g_5 &= \cosh(c\eta)\frac{\sinh(d\eta)}{d}, \quad g_6 = \sinh(c\eta)\cosh(d\eta),
\end{aligned} \tag{25}$$

the general solution is expressible as

$$\bar{\beta} = \sum_{i=1}^{6} g_i B_i, \quad \bar{\theta} = \sum_{i=1}^{6} g_i C_i, \quad \bar{W} = \sum_{i=1}^{6} g_i A_i. \tag{26}$$

We remark that this solution form is quite general, and incorporates in particular the cases of equal roots ($d = 0$) and unequal roots ($d =$ purely imaginary), provided we set $\sinh(d\eta)/d = \eta$ in the former case and use $\sinh(d\eta)/d = \sin(d\eta)/d$ and $\cosh(d\eta) = \cos(d\eta)$ in the latter case. In matrix form the solution can be expressed as

$$\bar{\beta} = \langle g\rangle\{B\}, \quad \bar{\theta} = \langle g\rangle\{C\}, \quad \bar{W} = \langle g\rangle\{A\}, \tag{27}$$

where $\langle g\rangle \equiv \langle g_1 g_2 g_3 g_4 g_5 g_6\rangle$. The derivatives are expressible as

$$\frac{\mathrm{d}\bar{\beta}}{\mathrm{d}\eta} = \langle g\rangle[Z]\{B\}, \quad \frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\eta} = \langle g\rangle[Z]\{C\}, \quad \frac{\mathrm{d}\bar{W}}{\mathrm{d}\eta} = \langle g\rangle[Z]\{A\}, \tag{28}$$

$$\frac{\mathrm{d}^2\bar{\beta}}{\mathrm{d}\eta^2} = \langle g\rangle[Z]^2\{B\}, \quad \frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}\eta^2} = \langle g\rangle[Z]^2\{C\}, \quad \frac{\mathrm{d}^2\bar{W}}{\mathrm{d}\eta^2} = \langle g\rangle[Z]^2\{A\}, \tag{29}$$

where

$$[Z] = \begin{bmatrix} 0 & -q^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & d^2 \\ 0 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & c & d^2 & 0 & 0 \\ 0 & 0 & 1 & c & 0 & 0 \end{bmatrix}. \tag{30}$$

Now, the connection between $\{B\}$, $\{C\}$ and $\{A\}$ is obtained by substituting eqns (27)–(29) in two of the governing equations (17), chosen here to be the first two. This gives

$$\begin{aligned}
&\left[\left(\frac{\varepsilon B}{F}+\frac{1}{h^2\alpha^2}\right)[I]-\varepsilon[Z]^2\right]\{B\}-\varepsilon\left[\frac{C+F}{F}\right][Z]\{C\}+\left[-\frac{B}{F}[I]+\left(2+\frac{C}{F}\right)[Z]^2\right]\{A\}=0, \\
&\varepsilon\left[\frac{C+F}{F}\right][Z]\{B\}+\left[-\frac{\varepsilon D}{F}[Z]^2+\left(\varepsilon+\frac{1}{h^2\alpha^2}\right)[I]\right]\{C\}+\left[\frac{D}{F}[Z]^3-\left(2+\frac{C}{F}\right)[Z]\right]\{A\}=0,
\end{aligned} \tag{31}$$

where $[I]$ denotes the identity matrix.

We write the above two equations as

$$[M]\{B\}-\varepsilon\left(\frac{C+F}{F}\right)[Z]\{C\}=[P]\{A\},$$

$$\varepsilon\left(\frac{C+F}{F}\right)[Z]\{B\}+[N]\{C\}=[Q]\{A\}, \tag{32}$$

by defining

$$[M]=\left[\left(\frac{\varepsilon B}{F}+\frac{1}{h^2\alpha^2}\right)[I]-\varepsilon[Z]^2\right], \tag{33}$$

$$[P]=\left[\frac{B}{F}[I]-\left(2+\frac{C}{F}\right)[Z]^2\right], \tag{34}$$

$$[N]=\left[-\frac{\varepsilon D}{F}[Z]^2+\left(\varepsilon+\frac{1}{h^2\alpha^2}\right)[I]\right], \tag{35}$$

$$[Q]=\left[-\frac{D}{F}[Z]^3+\left(2+\frac{C}{F}\right)[Z]\right]. \tag{36}$$

The desired relationships are then given by

$$\{B\}=[S]\{A\},\quad \{C\}=[T]\{A\} \tag{37}$$

in which we have introduced the notation

$$[T]=\left[\varepsilon^2\left(\frac{C+F}{F}\right)^2[Z][M]^{-1}[Z]+[N]\right]^{-1}\left[[Q]-\varepsilon\left(\frac{C+F}{F}\right)[Z][M]^{-1}[P]\right], \tag{38}$$

$$[S]=\varepsilon\left(\frac{C+F}{F}\right)[M]^{-1}[Z][T]+[M]^{-1}[P]. \tag{39}$$

The general solution and its derivatives, eqns (27)–(28), can now be written as

$$\bar{\beta}=\langle g\rangle[S]\{A\},\quad \bar{\theta}=\langle g\rangle[T]\{A\},\quad \bar{W}=\langle g\rangle\{A\}, \tag{40}$$

$$\frac{\mathrm{d}\bar{\beta}}{\mathrm{d}\eta}=\langle g\rangle[Z][S]\{A\}=\langle g\rangle[\hat{S}]\{A\},\quad \frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\eta}=\langle g\rangle[Z][T]\{A\}=\langle g\rangle[\hat{T}]\{A\}$$

$$\frac{\mathrm{d}\bar{W}}{\mathrm{d}\eta}=\langle g\rangle[Z]\{A\}, \tag{41}$$

where we have put

$$[\mathring{S}] = [Z][S] \quad \text{and} \quad [\mathring{T}] = [Z][T]. \tag{42}$$

The constants $\{A\}$ are to be determined by requiring the solution to satisfy the boundary conditions.

Boundary conditions

The solution obtained above can be used to find elastic/plastic buckling loads of rectangular plates for a variety of longitudinal edge conditons, keeping in mind the provision that the loaded edges are always simply supported (satisfying $\psi = 0$ rather than $M_{xy} = 0$). As remarked in the Introduction, we choose to treat here only the cases of (i) plates simply supported on all sides, and (ii) those simply supported on three sides but free on the fourth (longitudinal) side. The boundary conditions required in treating them can be listed as follows:

$$w = 0, \quad Q_y = 0, \quad M_{yy} = 0, \quad \phi = 0, \quad M_{xy} = 0 \tag{43}$$

at the longitudinal (η = constant) edges. In terms of the solution functions these can be expressed as

$$w = 0 \rightarrow \bar{W} = 0 \rightarrow \langle \bar{g} \rangle [I]\{A\} = 0, \tag{44}$$

$$Q_y = 0 \rightarrow \bar{\theta} = 0 \rightarrow \langle \bar{g} \rangle [T]\{A\} = 0, \tag{45}$$

$$M_{yy} = 0 \rightarrow \varepsilon D\bar{\theta}_{,\eta} - D\bar{W}_{,\eta\eta} - \varepsilon C\bar{\beta} + C\bar{W} = \langle \bar{g} \rangle \left[\varepsilon[\mathring{T}] - [Z]^2 - \varepsilon \frac{C}{D}[S] + \frac{C}{D}[I] \right]\{A\} = \langle \bar{g} \rangle [\lambda]\{A\} = 0, \tag{46}$$

$$\phi = 0 \rightarrow \Phi = \langle \bar{g} \rangle [-\varepsilon[S] + [I]]\{A\} = \langle \bar{g} \rangle [\mu]\{A\} = 0. \tag{47}$$

$$M_{xy} = 0 \rightarrow \varepsilon(\bar{\theta} + \bar{\beta}_{,\eta}) - 2\bar{W}_{,\eta} = \langle \bar{g} \rangle [\varepsilon[T] + \varepsilon[\mathring{S}] - 2[Z]]\{A\} = \langle \bar{g} \rangle [\delta]\{A\} = 0, \tag{48}$$

where we recall that $\varepsilon = F/G$, and $\langle \bar{g} \rangle$ denotes values of the functions $\langle g \rangle$ at an η = constant boundary. The newly introduced matrices $[\lambda]$, $[\mu]$, and $[\delta]$ have the following structures:

$$[\lambda] = \varepsilon[\mathring{T}] - [Z]^2 - \varepsilon\frac{C}{D}[S] + \frac{C}{D}[I] = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & \lambda_4 d^2 & 0 & 0 \\ 0 & 0 & \lambda_4 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & \lambda_4 d^2 \\ 0 & 0 & 0 & 0 & \lambda_4 & \lambda_3 \end{bmatrix}, \tag{49}$$

$$[\mu] = -\varepsilon[S] + [I] = \begin{bmatrix} \mu_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_3 & \mu_4 d^2 & 0 & 0 \\ 0 & 0 & \mu_4 & \mu_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_3 & \mu_4 d^2 \\ 0 & 0 & 0 & 0 & \mu_4 & \mu_3 \end{bmatrix}, \tag{50}$$

$$[\delta] = \varepsilon[T]+\varepsilon[S]-2[Z] = \begin{bmatrix} 0 & -\delta_2 q^2 & 0 & 0 & 0 & 0 \\ \delta_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_3 & \delta_4 d^2 \\ 0 & 0 & 0 & 0 & \delta_4 & \delta_3 \\ 0 & 0 & \delta_3 & \delta_4 d^2 & 0 & 0 \\ 0 & 0 & \delta_4 & \delta_3 & 0 & 0 \end{bmatrix}. \quad (51)$$

wherein we have put $\lambda_2 = \mu_2 = \delta_1 = 0$ by virtue of the nature of $[Z]$.

4. RECTANGULAR SANDWICH PLATES SIMPLY SUPPORTED ON ALL SIDES

The coordinate system with respect to the plate configuration is shown in Fig. 1. According to the conventional (Kirchhoff) theory for homogeneous plates, the two boundary conditions which correspond to the simple-support condition at the edge y = constant are $M_{yy} = 0$ and $w = 0$. However, as indicated in Section 3, the present theory for sandwich plates requires the satisfaction of an additional condition, which can be either $M_{xy} = 0$ or $\Phi = 0$. Both conditions (or an intermediate one) can occur in practice, and the appropriate choice will be dictated by the physical features of the problem at hand. The analysis presented herein is carried out for both types of "simple supports" at the edges $y = \pm b/2$, although it may be recalled that the loaded edges ($x = 0, a$) have been assumed simply supported in the sense $M_{xx} = w = \psi = 0$. This latter choice was necessary for effecting separation of variables.

Since the edge conditions at $y = \pm b/2$ are symmetric, the plate can buckle either in a symmetric mode, implying $A_1 = A_5 = A_6 = 0$, or in an antisymmetric mode, meaning $A_2 = A_3 = A_4 = 0$. Hence, the characteristic determinant which is required to vanish for buckling is of 3×3 size.

(1) Boundary conditions case A: $\begin{Bmatrix} w \\ M_{yy} \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$ at $\eta = \pm \alpha b/2$.

These conditions imply that for a symmetric mode of buckling

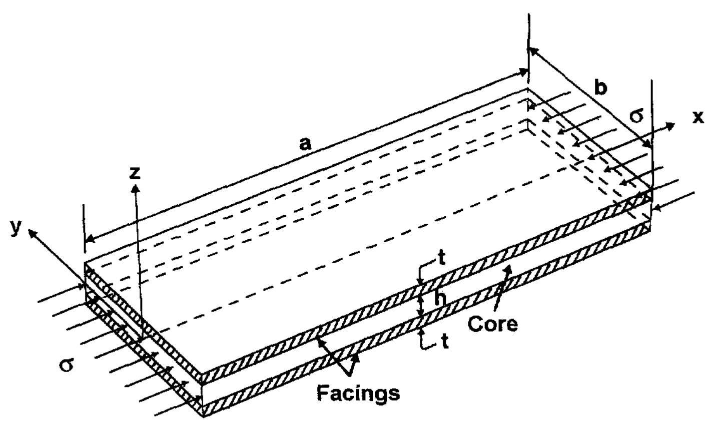

Fig. 1. Sandwich plate configuration and coordinate system.

$$\begin{bmatrix} \langle \bar{g} \rangle [I][H] \\ \langle \bar{g} \rangle [\lambda][H] \\ \langle \bar{g} \rangle [\mu][H] \end{bmatrix} \begin{Bmatrix} A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{52}$$

wherein $\langle \bar{g} \rangle$ denotes $\langle g \rangle$ evaluated at $\eta = \alpha b/2 = m\pi b/2a$, and

$$[H] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{53}$$

For a non-trivial solution (i.e. buckling), the determinant of the above set of homogeneous equations must vanish:

$$\det \begin{bmatrix} \bar{g}_2 & \bar{g}_3 & \bar{g}_4 \\ \lambda_1 \bar{g}_2 & \lambda_3 \bar{g}_3 + \lambda_4 \bar{g}_4 & d^2 \lambda_4 \bar{g}_3 + \lambda_3 \bar{g}_4 \\ \mu_1 \bar{g}_2 & \mu_3 \bar{g}_3 + \mu_4 \bar{g}_4 & d^2 \mu_4 \bar{g}_3 + \mu_3 \bar{g}_4 \end{bmatrix} = 0. \tag{54}$$

When expanded, this equation, determining the buckling parameter, becomes:

$$\Delta_1 \cos \frac{q\alpha b}{2} \left[\cosh^2 \left(\frac{c\alpha b}{2} \right) + \sinh^2 \left(\frac{d\alpha b}{2} \right) \right] = 0, \tag{55}$$

where

$$\Delta_1 = [\lambda_4(\mu_3 - \mu_1) - \mu_4(\lambda_3 - \lambda_1)]. \tag{56}$$

Considering now the antisymmetric mode of buckling, requiring $A_2 = A_3 = A_4 = 0$, the characteristic equation is found to be of the following form:

$$\Delta_1 \sin \frac{q\alpha b}{2} \left[\sinh^2 \left(\frac{c\alpha b}{2} \right) - \sinh^2 \left(\frac{d\alpha b}{2} \right) \right] = 0, \tag{57}$$

wherein Δ_1 is exactly that given above for the symmetric buckling mode equation (56). Since the mode can either be symmetric or antisymmetric, Δ_1 does not vanish, and accordingly for buckling we must have

$$\cos \frac{q\alpha b}{2} = 0 \text{ or } \sin \frac{q\alpha b}{2} = 0 \tag{58}$$

for symmetric and antisymmetric modes, respectively. This implies that

$$q = \frac{n\pi}{\alpha b} = \frac{na}{mb}, \tag{59}$$

where n is a positive integer equal to the number of half waves in the y-direction, being odd for symmetric modes and even for antisymmetric ones. On the other hand, m is, as may be recalled, the number of half-waves in the longitudinal direction. Now since $-q^2$ is a root of eqn (19), the above value can be used to yield the critical load in the closed form expression as

$$P_{cr} = D\left(\frac{n\pi}{b}\right)^2\left(\frac{na}{mb}\right)^2$$

$$\cdot\frac{1+\left(\frac{4F+2C}{D}\right)\left(\frac{mb}{na}\right)^2+\frac{B}{D}\left(\frac{mb}{na}\right)^4+\varepsilon\left(\frac{nh\pi}{b}\right)^2\left[1+(1-R)\left(\frac{mb}{na}\right)^2+\left(\frac{B}{D}-R\right)\left(\frac{mb}{na}\right)^4+\frac{B}{D}\left(\frac{mb}{na}\right)^6\right]}{1+\varepsilon\left(\frac{nh\pi}{b}\right)^2\left\{1+\frac{D}{F}+\left(1+\frac{B}{F}\right)\left(\frac{mb}{na}\right)^2\right\}+\varepsilon^2\left(\frac{nh\pi}{b}\right)^4\left[\frac{D}{F}+\frac{B}{F}\left(\frac{mb}{na}\right)^4-\frac{RD}{F}\left(\frac{mb}{na}\right)^2\right]}. \tag{60}$$

This type of result is classical, and has been given, for example, by Seide and Stowell (1948). From eqn (54) it follows that for the symmetric mode shape only A_2, B_2 and C_1 are non-zero, and that for the antisymmetric mode only A_1, B_1, C_2 are non-zero. In other words, the modes are purely sinusoidal.

(2) Boundary conditions case B: $\begin{Bmatrix} w \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$ at $y = \pm b/2$.

This is the case of simple supports without any rotation restraint. For symmetric modes, the above boundary conditions are expressible as

$$\begin{bmatrix} \langle \bar{g}\rangle[I][H] \\ \langle \bar{g}\rangle[\lambda][H] \\ \langle \bar{g}\rangle[\delta][H] \end{bmatrix}\begin{Bmatrix} A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{61}$$

where $\langle \bar{g}\rangle$ and $[H]$ are the same as in case A. The buckling condition can be written as

$$\det\begin{bmatrix} \bar{g}_2 & \bar{g}_3 & \bar{g}_4 \\ \lambda_1\bar{g}_2 & \lambda_3\bar{g}_3+\lambda_4\bar{g}_4 & d^2\lambda_4\bar{g}_3+\lambda_3\bar{g}_4 \\ -q^2\delta_2\bar{g}_1 & \delta_3\bar{g}_5+\delta_4\bar{g}_6 & d^2\delta_4\bar{g}_5+\delta_3\bar{g}_6 \end{bmatrix} = 0, \tag{62}$$

which in expanded form is

$$\Gamma_1\frac{1}{q}\sin\left(\frac{qm\pi b}{2a}\right)+\cos\left(\frac{qm\pi b}{2a}\right)\left[\Gamma_2\sinh\left(\frac{cm\pi b}{2a}\right)\cosh\left(\frac{cm\pi b}{2a}\right)-\Gamma_3\frac{1}{d}\sinh\left(\frac{dm\pi b}{2a}\right)\cosh\left(\frac{dm\pi b}{2a}\right)\right]=0 \tag{63}$$

where

$$\Gamma_1 = q^2\delta_2\lambda_4\left[\cosh^2\left(\frac{cm\pi b}{2a}\right)+\sinh^2\left(\frac{dm\pi b}{2a}\right)\right],$$

$$\Gamma_2 = (\lambda_4\delta_3-\lambda_3\delta_4+\lambda_1\delta_4),\quad \Gamma_3 = (\lambda_3\delta_3-d^2\lambda_4\delta_4-\lambda_1\delta_3). \tag{64}$$

On the other hand, the characteristic equation for antisymmetric modes is found to be

$$\Gamma_1^* \cos\left(\frac{qm\pi b}{2a}\right) + \frac{1}{q}\sin\left(\frac{qm\pi b}{2a}\right)\left[\Gamma_2 \sinh\left(\frac{cm\pi b}{2a}\right)\cosh\left(\frac{cm\pi b}{2a}\right) + \Gamma_3 \frac{1}{d}\sinh\left(\frac{dm\pi b}{2a}\right)\cosh\left(\frac{dm\pi b}{2a}\right)\right] = 0, \quad (65)$$

where

$$\Gamma_1^* = -\delta_2\lambda_4\left[\sinh^2\left(\frac{cm\pi b}{2a}\right) - \sinh^2\left(\frac{dm\pi b}{2a}\right)\right] \quad (66)$$

and Γ_2 and Γ_3 are exactly those for the symmetric modes.

Numerical results and discussion

In view of the complexity of the characteristic equations, a trial and error numerical approach using a computer becomes essential for obtaining the buckling loads. For this purpose the facings are taken to be of aluminum (24S-T3) alloy, having a uniaxial stress–strain ($\varepsilon_1 - \sigma_1$) curve, modeled by

$$\varepsilon_1 = \frac{\sigma_1}{11100} + 0.002\left(\frac{\sigma_1}{43.5}\right)^7. \quad (67)$$

The above formula models the stress–strain curve used by Seide and Stowell (1948), and is meaningful up to a stress level of about 45 ksi (310 MPa) with plasticity setting around 25 ksi (172 MPa). Poisson's ratio is taken as 1/3. The core is taken to be balsa wood, idealized as an isotropic elastic material with Young's modulus equal to 53.2 ksi (367 MPa) and Poisson's ratio equal to 0.4. This also is the same material as used by Seide and Stowell (1948). The core is assumed to remain elastic.

Choosing specific values of t/h, a/b, b/h and a provisional value of m (the number of half-waves in which the plate could buckle in the longitudinal direction), the computer program determines the minimum shortening strain (and hence σ_f, σ_c and P_{cr}) for which the characteristic equation is satisfied. The procedure is repeated for different values of m; the critical load P_{cr} is given by that m for which it is the smallest.

Figures 2 and 3 show the variation of the critical buckling stress with respect to the b/h ratio of the plate. This buckling stress is defined nominally as $\sigma_{cr} = P_{cr}/2t = \sigma_f + (h/2t)\sigma_c$; it will be close to the buckling stress in the facing material if the Young's modulus of the core material is much smaller than that of the facing material.

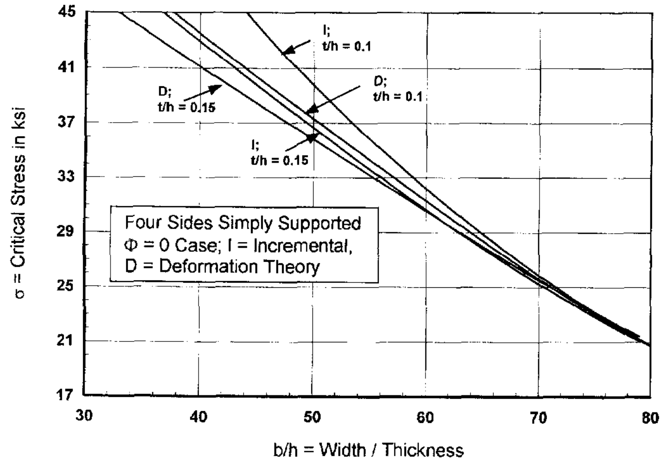

Fig. 2. Critical stress σ_{cr} vs b/h for buckling of long rectangular sandwich plates ($a/b = 10$) simply supported on all four sides with $\phi = 0$ assumed on the unloaded edges (case A).

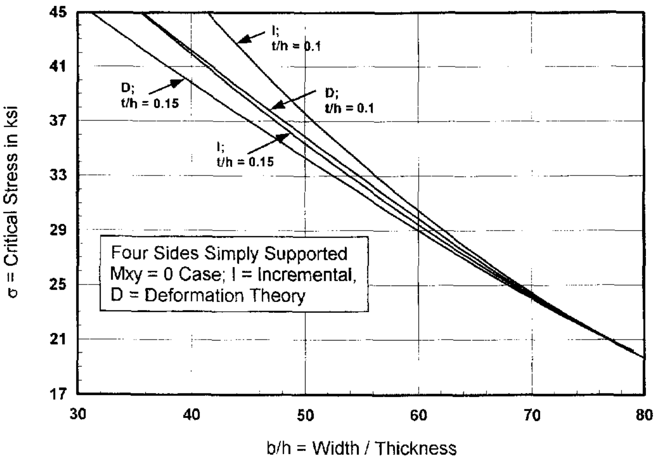

Fig. 3. Critical stress σ_{cr} vs b/h for buckling of long rectangular sandwich plates ($a/b = 10$) simply supported on all four sides with $M_{xy} = 0$ assumed on the unloaded edges (case B).

The plate dimensions for which the buckling graphs are plotted are $a/b = 10$ and $t/h = 0.15$ and 0.1. This a/b ratio is considered large enough to render the plate "infinitely" long. Figure 2 shows the buckling curve for the case when simple supports on the $y = \pm b/2$ edges are assumed to mean $\phi = 0$ (together with $w = M_{yy} = 0$). Results from both the incremental and deformation theories of plasticity are shown. As usual, the results from the incremental theory correspond to higher buckling loads than those from the deformation theory. This difference becomes greater at higher buckling loads, reaching about 10% for $t/h = 0.1$ and about 5% for $t/h = 0.15$, at the upper limit of the meaningful range of the stress–strain curve. Figure 3 gives the buckling load when the condition of simple supports is modeled by taking $w = M_{yy} = 0$ and $M_{xy} = 0$ (rather than $\phi = 0$). The differences between the results of the two theories are found similar to those for Fig. 2. However, comparing Figs 2 and 3, it is seen that the buckling loads in the $M_{xy} = 0$ case are about 5% lower than in the case $\phi = 0$, for both plasticity theories.

Mode shapes

The mode shapes in the longitudinal directions are sinusoidal. In the transverse direction the mode shapes for displacement and rotation can be expressed in terms of the vectors $\{A\}$, $\{B^*\}$, $\{C^*\}$, by choosing the dominant element in $\{A\}$ as unity. We have from eqns (15),

$$\bar{W} = \langle g\rangle\{A\}, \quad \bar{\phi} = \langle g\rangle\{B^*\}, \quad \bar{\psi} = \langle g\rangle\{C^*\}, \tag{68}$$

where

$$\{B^*\} = [[I]-\varepsilon[S]]\{A\} \quad \text{and} \quad \{C^*\} = [[Z]-\varepsilon[T]]\{A\}. \tag{69}$$

As a typical example of the relative magnitudes, Table 1 defines the mode shapes for the

Table 1. Buckling mode shapes of a sandwich plate simply supported on all four sides

	Deformation theory ($a/b = 10$, $b/h = 45$, $t/h = 0.1$)	
	$\phi = 0$ ($m = 14$, $\sigma_{cr} = 40.4$ ksi)	$M_{xy} = 0$ ($m = 13$, $\sigma_{cr} = 39.0$ ksi)
Roots	$q = 0.71$, $c = 2.30$, $d = 0.71$	$q = 0.73$, $c = 2.4$, $d = 0.82$
$\langle A\rangle$	$\langle 0, 1, 0, 0, 0, 0\rangle$	$\langle 0, 1, 0.0052, -0.0063, 0, 0\rangle$
$\langle B^*\rangle$	$\langle 0, 0.75, 0, 0, 0, 0\rangle$	$\langle 0, 0.76, 0.011, -0.011, 0, 0\rangle$
$\langle C^*\rangle$	$\langle -0.33, 0, 0, 0, 0, 0\rangle$	$\langle -0.37, 0, 0, 0, 0.013, -0.015\rangle$
	Incremental Theory ($a/b = 10$, $b/h = 45$, $t/h = 0.1$)	
	$\phi = 0$ ($m = 16$, $\sigma_{cr} = 44.1$ ksi)	$M_{xy} = 0$ ($m = 14$, $\sigma_{cr} = 41.8$ ksi)
Roots	$q = 0.62$, $c = 1.86$, $d = 0.36\,i$	$q = 0.67$, $c = 2.07$, $d = 0.31$
$\langle A\rangle$	$\langle 0, 1, 0, 0, 0, 0\rangle$	$\langle 0, 1, 0.0019, -0.0052, 0, 0\rangle$
$\langle B^*\rangle$	$\langle 0, 0.72, 0, 0, 0, 0\rangle$	$\langle 0, 0.74, 0.0088, -0.012, 0, 0\rangle$
$\langle C^*\rangle$	$\langle -0.21, 0, 0, 0, 0, 0\rangle$	$\langle -0.27, 0, 0, 0, 0.0075, -0.015\rangle$

two alternative types of simple supports at the edges $y = \pm b/2$, for a plate of size $a/b = 10$, $b/h = 45$, and $t/h = 0.1$. The mode shapes are necessarily sinusoidal for the $\phi = 0$ cases and are easy to visualize, while those for the $M_{xy} = 0$ cases are not too different from them. Clearly, the relative magnitudes are larger for the deformation theory than for the incremental one. Moreover, they are larger for the $M_{xy} = 0$ cases, indicating a more unconstrained behavior. Consistent with these observations, the number of buckles is larger for the incremental theory and $\phi = 0$ cases.

5. RECTANGULAR SANDWICH PLATES SIMPLY SUPPORTED ON THREE SIDES AND FREE ON THE FOURTH SIDE

It may be recalled that two of the simply supported edges are the transverse ones, $x = 0$, a, at which the compression is applied. For simplifying the analysis in this section, the origin of the coordinate system is changed so that the third simply supported edge is the longitudinal edge $y = 0$, while the free edge is $y = b$. The boundary conditions at the free ($y = b$) edge are clearly $M_{xy} = M_{xy} = Q = 0$; however those at the supported ($y = 0$) edge can be either $M_{yy} = w = M_{xy} = 0$ or $M_{yy} = w = \psi = 0$. As in Section 4, both of these possibilities are investigated as cases C and D.

(3) Boundary conditions case C: $\begin{Bmatrix} w \\ M_{yy} \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$ at $y = 0$ and $\begin{Bmatrix} M_{yy} \\ Q_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$ at $y = b$.

These boundary conditions amount to the buckling of a plate of width $2b$ in an antisymmetric mode about the centerline $y = 0$, meaning that $A_2 = A_3 = A_4 \equiv 0$ in the solution. This buckling is identifiable with that of a sandwich column of "cruciform" section. The characteristic determinant is of 3×3 size and arises from

$$\begin{bmatrix} \langle \bar{g} \rangle [\lambda][H] \\ \langle \bar{g} \rangle [T][H] \\ \langle \bar{g} \rangle [\delta][H] \end{bmatrix} \begin{Bmatrix} A_1 \\ A_5 \\ A_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{70}$$

where $\langle \bar{g} \rangle$ are values of $\langle g \rangle$ at $y = b$ (i.e. $\eta = m\pi b/a$), and

$$[H] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{71}$$

The resulting characteristic equation is:

$$\det \begin{bmatrix} \lambda_1 \bar{g}_1 & \lambda_3 \bar{g}_5 + \lambda_4 \bar{g}_6 & d^2 \lambda_4 \bar{g}_5 + \lambda_3 \bar{g}_6 \\ t_2 \bar{g}_2 & t_3 \bar{g}_3 + t_4 \bar{g}_4 & d^2 t_4 \bar{g}_3 + t_3 \bar{g}_4 \\ \delta_2 \bar{g}_2 & \delta_3 \bar{g}_3 + \delta_4 \bar{g}_4 & d^2 \delta_4 \bar{g}_3 + \delta_3 \bar{g}_4 \end{bmatrix} = 0, \tag{72}$$

which, when expanded, can be written as

$$\Gamma_1 \frac{1}{q}\sin\left(\frac{qm\pi b}{a}\right)+\cos\left(\frac{qm\pi b}{a}\right)\left[\Gamma_2 \sinh\left(\frac{cm\pi b}{a}\right)\cosh\left(\frac{cm\pi b}{a}\right)\right.$$
$$\left.+\Gamma_3 \frac{1}{d}\sinh\left(\frac{dm\pi b}{a}\right)\cosh\left(\frac{dm\pi b}{a}\right)\right] = 0, \quad (73)$$

where

$$\Gamma_1 = \lambda_1(\delta_3 t_4 - \delta_4 t_3)\left[\sinh^2\left(\frac{cm\pi b}{a}\right)+\cosh^2\left(\frac{dm\pi b}{a}\right)\right],$$
$$\Gamma_2 = [\lambda_4(\delta_2 t_3 - \delta_3 t_2) - \lambda_3(\delta_2 t_4 - \delta_4 t_2)],$$
$$\Gamma_3 = [\lambda_3(\delta_2 t_3 - \delta_3 t_2) - d^2\lambda_4(\delta_2 t_4 - \delta_4 t_2)]. \quad (74)$$

(4) Boundary conditions case D: $\begin{Bmatrix} w \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$ at $y = 0$ and $\begin{Bmatrix} M_{yy} \\ Q_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$ at $y = b$.

These equations can be written as

$$\begin{bmatrix} \langle \bar{g}^{\cdot} \rangle [I] \\ \langle \bar{g}^{\cdot} \rangle [\lambda] \\ \langle \bar{g}^{\cdot} \rangle [\delta] \\ \langle \bar{g}^{\dagger} \rangle [\lambda] \\ \langle \bar{g}^{\dagger} \rangle [T] \\ \langle \bar{g}^{\dagger} \rangle [\delta] \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (75)$$

where $\langle \bar{g}^{\cdot} \rangle$ and $\langle \bar{g}^{\dagger} \rangle$ denote $\langle g \rangle$ evaluated at $\eta = 0$ and $\eta = (m\pi b/a)$, respectively. The 6×6 set of equations can be reduced to a 3×3 set by using the fact that the first three equations give

$$A_4 = -A_2; \quad \lambda_4 A_3 = (\lambda_3 - \lambda_1)A_2; \quad \delta_2 A_1 = -\delta_4 A_5 - \delta_3 A_6. \quad (76)$$

In matrix notation the resulting 3×3 set (with redefined constants) is:

$$\begin{bmatrix} \langle \bar{g}^{\dagger} \rangle [\lambda][H] \\ \langle \bar{g}^{\dagger} \rangle [T][H] \\ \langle \bar{g}^{\dagger} \rangle [\delta][H] \end{bmatrix} \begin{Bmatrix} A_2 \\ A_5 \\ A_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (77)$$

where

$$[H] = \begin{bmatrix} 0 & -\delta_4 & -\delta_3 \\ \lambda_4 & 0 & 0 \\ \lambda_3 - \lambda_1 & 0 & 0 \\ -\lambda_4 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & \delta_2 \end{bmatrix}. \quad (78)$$

The characteristic determinant is now of 3×3 size which, if desired, can be expanded explicitly. For the sake of brevity, we refrain from writing it, and instead opt for its direct numerical evaluation.

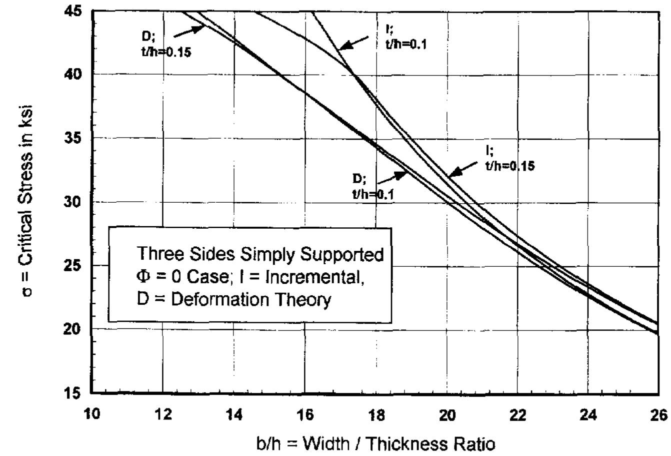

Fig. 4. Critical stress σ_{cr} vs b/h for buckling of long rectangular sandwich plates ($a/b = 10$) simply supported on three sides with $\phi = 0$ assumed on the unloaded supported edge (case C).

Numerical results and discussion

The material and plate dimensions are taken to be exactly the same as in Section 4. In particular, the plates are considered "infinitely" long ($a/b = 10$). Figure 4 shows variation of the (nominal) critical stress when the edge $y = 0$ is simply supported in the sense $\phi = 0$ (in addition to $M_{yy} = w = 0$). It can be seen that the buckling stresses obtained from the incremental theory are higher than those given by the deformation theory of plasticity. The difference increases to as much as 20% at the upper limit of the meaningful range of the stress–strain curve for $t/h = 0.1$. The difference is a bit smaller for $t/h = 0.15$.

Figure 5 shows the variation of buckling stress when the edge $y = 0$ is considered simply supported in the sense of $M_{xy} = 0$. Again, the results from the incremental theory are higher than those from the deformation theory, with relative differences similar to those found in Fig. 4. However, a comparison of Fig. 5 with Fig. 4 reveals the fact that, in the case of sandwich plates simply supported on three sides and free on the fourth side, the values of the buckling loads are considerably different for the two choices of simple supports on the edge $y = 0$; corresponding to $\phi = 0$ (Fig. 4) and corresponding to $M_{xy} = 0$ (Fig. 5). As an example, for a plate with $t/h = 0.15$ and using the incremental theory of plasticity, it is seen that the buckling stress for the $M_{xy} = 0$ case is generally 30% smaller than when $\phi = 0$ is imposed. Similarly, for the deformation theory, the decrease is again significant but somewhat smaller, around 23%. We recall that such a reduction in the buckling stress

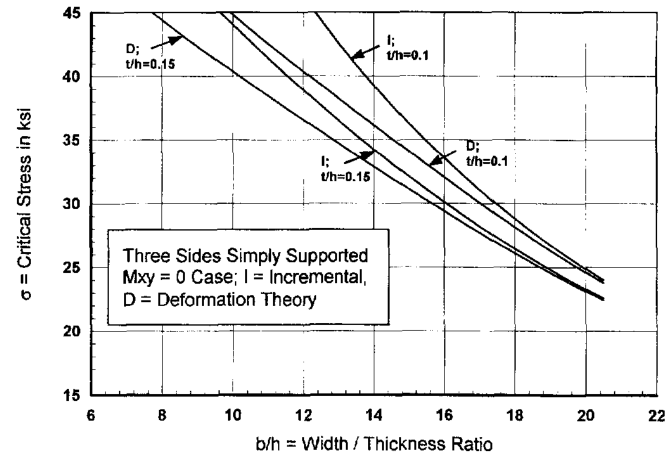

Fig. 5. Critical stress σ_{cr} vs b/h for buckling of long rectangular sandwich plates ($a/b = 10$) simply supported on three sides with $M_{xy} = 0$ assumed on the unloaded supported edge (case D).

is not predicted for plates simply supported on four sides (dealt with in Section 4), when the $\phi = 0$ condition is changed to $M_{xy} = 0$ at the longitudinal edges, $y = \pm b/2$.

It may also be noted that, in the case of homogeneous plates (in contrast to the sandwich plate), the decrease in the buckling stress when $\phi = 0$ is changed to $M_{xy} = 0$ is at most around 10% (Shrivastava, 1979), whether the plate is simply supported on all four sides or is simply supported on three sides. When the present theory is specialized to homogeneous plates, the computer program gives results for the cases $\phi = 0$ and $M_{xy} = 0$, which are within the 10% difference and which match with those obtained from the application of the theory in Shrivastava (1979). Thus, the accuracy of the present theory and computer programs has been verified.

It may therefore be concluded that this considerable and rather unexpected difference in the buckling stress given by Figs 4 and 5 is peculiar to the sandwich plate. This result, to the author's knowledge, has not been previously reported, even for the elastic buckling of sandwich plates. It underscores the importance of distinguishing between the alternative specifications of simple supports in sandwich plates, $M_{xy} = 0$ and $\phi = 0$ (or $\psi = 0$). It appears, however, that significant differences arise only when the boundary conditions on the opposite edges are not symmetric. In the case of plates simply supported on all four sides, dealt with in Section 4, the boundary conditions are symmetric and the relaxation of the boundary conditions from $\phi = 0$ to $M_{xy} = 0$ on the longitudinal edges does not result in a large reduction of the buckling load.

The question as to what happens when the boundary conditions are changed from $\psi = 0$ to $M_{xy} = 0$ on the loaded edges, $x = 0, a$, cannot be answered precisely by the present analysis. An analysis, taking such boundary conditions into account, will have to be numerical from the very beginning, employing either the finite element or the finite difference method. It may, however, be conjectured that, since these boundary conditions are symmetric, the relaxation from $\psi = 0$ to $M_{xy} = 0$ at $x = 0$, a, edges may not produce a reduction in the buckling stress which is more than 10%, as was the case when $\phi = 0$ was changed to $M_{xy} = 0$ on the edges $y = \pm b/2$ in Section 4.

Mode shapes

Table 2 specifies the numerical values for determining the transverse mode shapes for a plate with dimensions $a/b = 10$, $b/h = 16$, and $t/h = 0.15$ for the two cases of boundary conditions, and for the two plasticity theories. The critical loads correspond to the plate buckling in a single half-wave (except for the incremental theory $\phi = 0$ case). In view of their conservative nature, we limit the following discussion to the deformation theory results. The mode shapes, shown in Figs 6 and 7, were obtained by using the Mathematica program of Wolfram Research Inc., Champaign, IL.

Figure 6 (a) shows the deformed shapes of the two faces due to buckling, when $\phi = 0$ is enforced at the supported longitudinal edge. As required by the theory, the in-plane deformations are exactly equal but opposite for the two facings. Figure 6 (b) shows the out-of-plane displacements of the buckled plate at various locations along the length. These displacements are similar to the torsional-mode buckling of a homogeneous cruciform

Table 2. Buckling mode shapes of a sandwich plate simply supported on three sides and free on the fourth

	Deformation theory ($a/b = 10$, $b/h = 16$, $t/h = 0.15$)	
	$\phi = 0$ ($m = 1$, $\sigma_{cr} = 38.6$ ksi)	$M_{xy} = 0$ ($m = 1$, $\sigma_{cr} = 29.4$ ksi)
Roots	$q = 2.16$, $c = 7.30$, $d = 4.81$	$q = 1.90$, $c = 6.72$, $d = 4.43$
$\langle A \rangle$	$\langle -0.78, 0, 0, 0, 1, -0.21 \rangle$	$\langle -0.64, -0.014, -0.064, 0.014, 1, -0.22 \rangle$
$\langle B^* \rangle$	$\langle 0.69, 0, 0, 0, 1.19, -0.23 \rangle$	$\langle -0.58, -0.013, -0.93, -0.18, 2.03, -0.048 \rangle$
$\langle C^* \rangle$	$\langle 0, -0.69, 2.80, -0.58, 0, 0 \rangle$	$\langle 0.046, -0.58, 2.62, -0.55, -0.24, 0.019 \rangle$
	Incremental theory ($a/b = 10$, $b/h = 16$, $t/h = 0.15$)	
	$\phi = 0$ ($m = 7$, $\sigma_{cr} = 42.8$ ksi)	$M_{xy} = 0$ ($m = 1$, $\sigma_{cr} = 30.1$ ksi)
Roots	$q = 0.56$, $c = 1.33$, $d = 0.60\,i$	$q = 1.89$, $c = 6.58$, $d = 4.30$
$\langle A \rangle$	$\langle 1, 0, 0, 0, 0.022, -0.11 \rangle$	$\langle -0.66, -0.015, -0.066, 0.015, 1, -0.23 \rangle$
$\langle B^* \rangle$	$\langle 0.55, 0, 0, 0, -0.033, 0.14 \rangle$	$\langle -0.59, -0.014, -0.94, -0.18, 2.06, -0.047 \rangle$
$\langle C^* \rangle$	$\langle 0, 0.26, 0.034, 0.12, 0, 0 \rangle$	$\langle 0.049, -0.59, 2.63, -0.57, -0.25, 0.019 \rangle$

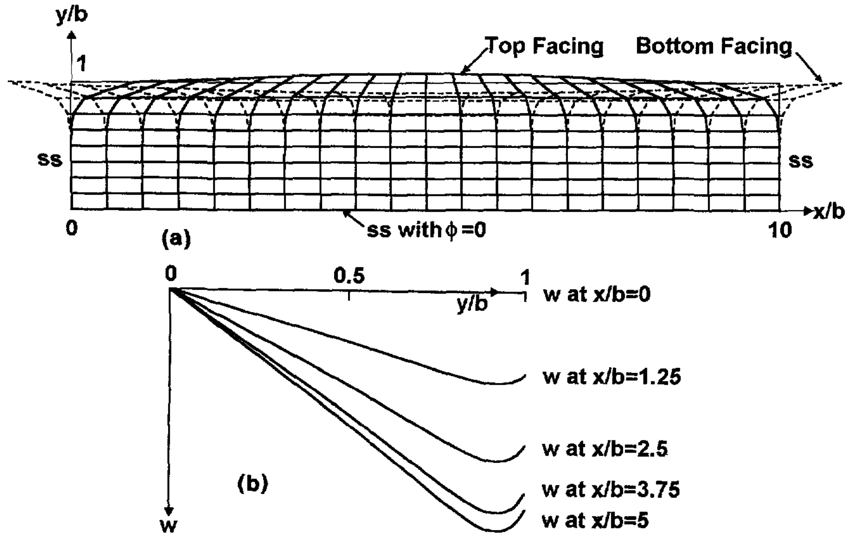

Fig. 6. Buckling mode shapes for a sandwich plate ($a/b = 10$, $b/h = 16$, $t/h = 0.15$) simply supported on three sides with $\phi = 0$ on the unloaded supported edge (case C, deformation theory). (a) In-plane displacements of the facings, (b) out-of-plane displacements.

column, except that there is significant bending at the free edge and the maximum deflection is somewhat inside it. We also observe that the bending is localized in the vicinity of the free edge; the facings are strained mostly in this area.

In contrast, the mode shapes of the $M_{xy} = 0$ case are as shown in Fig. 7. Here, the in-plane displacements for the two faces, Fig. 7 (a), are quite different in character from those in Fig. 6 (a). Each facing plate is able to buckle somewhat independently in the "torsional-flexural" mode. Moreover, the bending-related displacements of the facings are no longer confined to the free edge. The mode of out-of-plane displacements, Fig. 7(b), is now very similar to the torsional mode of a cruciform column, with virtually no bending across the

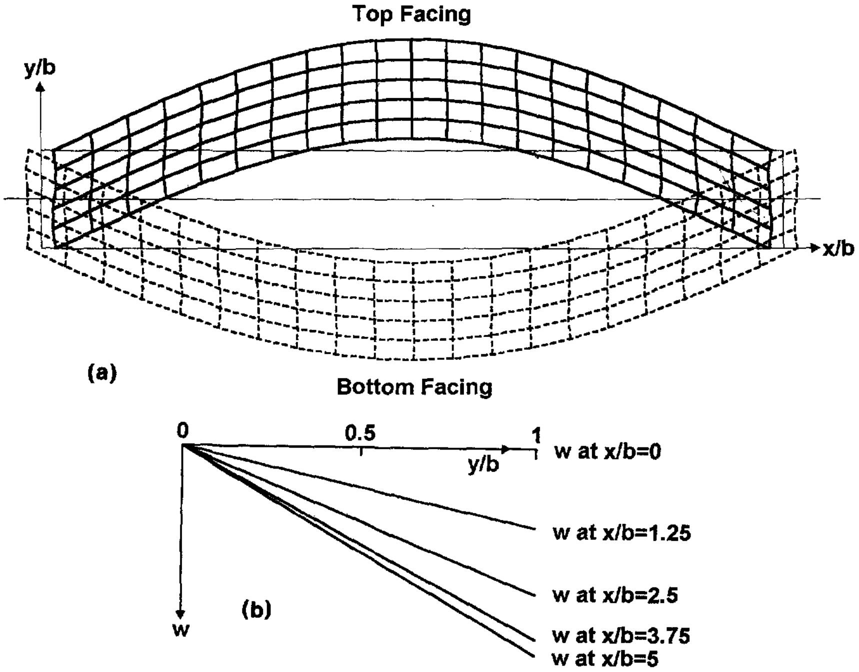

Fig. 7. Buckling mode shapes for the same plate as Fig. 6, but with $M_{xy} = 0$ on the unloaded supported edge (case D, deformation theory). (a) In-plane displacements of the two facings, (b) out-of-plane displacements.

width of the plate. In essence, the particular combination of boundary conditions $M_{yy} = 0$ on $x = 0, a$, and $M_{xy} = 0$ on $y = 0, b$, has made it possible for the facings to undergo rather unconstrained displacements (i.e. the ϕ and ψ rotations), by allowing the low modulus core to deform both in in-plane and out-of-plane shears. Calculations show that this observation remains valid even for a relatively short (say $a/b = 1$) plate. Thus, it is mainly a boundary condition phenomenon and can be expected to occur for sandwich plates in elastic as well as plastic regimes. However, as mentioned before, this mode is not a critical one when the boundary conditions on the rotation ϕ are symmetric, as in the case of the simply supported plate on four sides with $M_{yy} = 0$ at $x = 0, a$ and $M_{xy} = 0$ at $y = \pm b/2$.

6. CONCLUSION

The analysis presented in this paper provides a quite straightforward method of calculating the buckling load of sandwich plates in both elastic and plastic regimes. However, due to a lack of experimental data, comparison between the theoretical results obtained in this paper and experimental results cannot be made. Nevertheless, part of the theoretical results contained in Section 4 are compared with those obtained by Seide and Stowell (1948), using a different approach. The comparison shown in Table 3 is based on the deformation theory of plasticity for infinitely long plates simply supported on four sides. The core is assumed to remain elastic.

With reference to Table 3, the agreement between the results of Seide and Stowell (1948) and those for the $\phi = 0$ case of the present theory is quite good, with differences not higher than +1.8% at the high end of the buckling stress, and not lower than −2.8% at the low end. On the other hand, the results obtained for $M_{xy} = 0$ of the present theory are all lower than those of Seide and Stowell (1948), by amounts from −1.7 to −8.0%. According to Seide and Stowell (1948), their theoretical results are in fair agreement with their experimental results, with an average difference of about 8% on the unconservative side. Hence, on the basis of these experimental results, it may be concluded that the buckling loads obtained in this paper for the $M_{xy} = 0$ case will be on average about 3% higher than the experimental results. This comparison confirms the validity of the analysis presented in this paper.

None of the results presented in Section 5 for the buckling of a plate simply supported on three sides and free on the fourth side have a counterpart in the paper of Seide and Stowell (1948). These results are entirely new, and are significant not only from a theoretical viewpoint, but also from a practical one. A lack of proper reinforcement at the edges of a

Table 3. Buckling stress in ksi (1 ksi = 6.89 MPa) for rectangular sandwich plates supported on all four sides; I = incremental, D = deformation theory; numbers in parenthesis indicate the half-waves in the longitudinal direction

$t/h = 0.15$	b/h = 40	50	60
$\phi = 0$	I: 44.0 (16)	I: 37.3 (15)	I: 31.2 (13)
(present theory)	D: 41.1 (16)	D: 35.8 (14)	D: 30.6 (13)
$M_{xy} = 0$	I: 41.9 (16)	I: 35.4 (14)	I: 29.4 (12)
(present theory)	D: 39.8 (15)	D: 34.3 (13)	D: 29.0 (12)
Seide and Stowell	D: 40.5	D: 36.0	D: 31.2
$t/h = 0.1$	b/h = 45	50	60
$\phi = 0$	I: 44.1 (16)	I: 39.8 (14)	I: 32.2 (12)
(present theory)	D: 40.4 (14)	D: 37.3 (13)	D: 31.3 (12)
$M_{xy} = 0$	I: 41.7 (14)	I: 37.6 (13)	I: 30.5 (12)
(present theory)	D: 39.0 (13)	D: 35.9 (12)	D: 29.9 (11)
Seide and Stowell	D: 39.7	D: 37.0	D: 32.5

Table 4. Buckling stress in ksi (1 ksi = 6.89 MPa) for rectangular sandwich plates simply supported on three sides and free on the fourth side; the number of half-waves is 1 for each of the listed cases

Incremental theory ($t/h = 0.1$)	b/h 17	19	21
$\phi = 0$ at $y = 0$	41.5	34.4	28.9
$M_{xy} = 0$ at $y = 0$	31.1	26.8	23.2
Deformation theory ($t/h = 0.1$)	**b/h 13**	**17**	**21**
$\phi = 0$ at $y = 0$	44.9	36.5	28.1
$M_{xy} = 0$ at $y = 0$	38.2	30.1	23.0

sandwich plate may cause the facings to buckle independently. Table 4 illustrates the drop in buckling stress from the $\phi = 0$ case to the $M_{xy} = 0$ case for plates with $t/h = 0.1$, $a/b = 10$, and some selected b/h ratios. The drop is more for the incremental theory (20–25%) than for the deformation theory (15–18%).

To assess some drawbacks, we recall the fact that the boundary conditions at the loaded edges can only be simply supported in the sense $\psi = 0$ if separation of variables is sought. For other boundary conditions, the governing equations will have to be solved numerically using either the finite difference or the finite element method. It may be argued, however, that since the boundary conditions are symmetric at $x = 0$, a, relaxation of the conditions from $\psi = 0$ to $M_{xy} = 0$ will probably not cause a decrease in the buckling stresses by more than 5–10% of their present computed values. Another deficiency is that wrinkling, which involves buckling of the two facings independently in small waves or ripples, cannot be dealt with by the present analysis, This kind of buckling is possible if the facings are too thin or the core is too soft.

Finally, one again encounters the situation that the bifurcation buckling stresses predicted on the basis of the incremental theory of plasticity are consistently higher than those predicted by using the deformation theory, as can be seen from Tables 3 and 4. The differences are, however, less pronounced compared to those for homogeneous plates.

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